

□ 1. Methods of Estimation

□ Classical Estimation Techniques

- **Method of Moments:** Equate sample moments to population moments to solve for parameters.
- **Least Squares:** Minimizes the sum of squared differences between observed and predicted values.
- **Maximum Likelihood Estimation (MLE):** Maximizes the likelihood function; properties include consistency, efficiency, and asymptotic normality.

□ Bayesian Estimation

- **Bayes' Method:** Combines prior beliefs with data via Bayes' theorem.
- **Prior & Posterior Distributions:** Prior reflects belief before data; posterior updates belief after data.
- **Bayes Estimators:** Minimize expected loss; depend on chosen loss function.
- **Minimax Estimator:** Minimizes the maximum risk across all parameter values.

□ Estimator Properties

- **Mean Square Error (MSE):** Combines bias and variance.
- **Consistency:** Estimator converges to true value as sample size increases.
- **BAN Estimators:** Best Asymptotically Normal estimators.
- **Loss & Risk Functions:** Quantify estimation error and expected loss.

□ 2. Sufficiency & Unbiased Estimation

□ Sufficiency

- **Sufficient Statistic:** Contains all information about the parameter.
- **Factorization Criterion:** Used to identify sufficient statistics.
- **Rao-Blackwell Theorem:** Improves an unbiased estimator using sufficiency.
- **Minimal Sufficient Statistic:** No smaller statistic is sufficient.

- **Exponential Family:** Distributions with natural sufficient statistics.

□ Unbiased Estimation

- **Unbiased Estimator:** Expected value equals true parameter.
- **Lower Bound for Variance:** Cramér-Rao Lower Bound.
- **Completeness:** No non-zero function of statistic has zero expectation.
- **Sufficiency + Completeness:** Guarantees uniqueness of unbiased estimator.

□ 3. Confidence Intervals

□ Basics

- **Definition:** Range likely to contain the true parameter.
- **Pivotal Quantity:** Function of data and parameter with known distribution.

□ Normal Distribution Sampling

- **Mean:** Use z or t-distribution depending on known/unknown variance.
- **Variance:** Use chi-square distribution.
- **Proportion:** Use normal approximation for large samples.
- **Simultaneous Intervals:** Bonferroni or multivariate methods.
- **Difference of Means:** Use pooled or separate variance methods.

□ Methods

- **Pivotal Quantity Method:** Construct interval using known distribution.
- **Statistical Method:** Use sample statistics and distribution.
- **Large Sample Intervals:** Use CLT for approximate intervals.
- **Bayesian Interval Estimates:** Credible intervals from posterior distribution.

□ Interpretation

- **Frequentist:** Probability refers to long-run frequency.
- **Bayesian:** Probability reflects belief about parameter.

□ 4. Hypothesis Testing: Foundations

□ Key Concepts

- **Population vs Sample:** Inference from sample to population.
- **Test Statistic:** Summary of data used for decision.
- **Critical Region:** Values leading to rejection of null.
- **Type I Error (α):** Rejecting true null.
- **Type II Error (β):** Failing to reject false null.
- **Power:** Probability of correctly rejecting false null.
- **Connection to Confidence Intervals:** Duality between tests and intervals.

□ Hypothesis Types

- **Simple vs Composite:** One value vs range of values.
- **Null (H_0) vs Alternative (H_1):** Status quo vs new claim.

□ 5. Hypothesis Testing: Methods

□ Optimal Tests

- **Most Powerful Test:** Maximizes power for given α .
- **Neyman-Pearson Lemma:** Foundation for MP tests.

□ Distributions

- **Normal, Chi-square, t, F:** Used for various tests.
- **Central Limit Theorem (CLT):** Justifies normal approximation.

□ Generalized Likelihood Ratio Test (GLRT)

- **Definition:** Ratio of maximized likelihoods under H_0 and H_1 .
- **Asymptotic Distribution:** Often chi-square.

□ Other Test Types

- **UMP Tests:** Uniformly most powerful across all alternatives.
- **Unbiased Tests:** Expected rejection rate under $H_1 \geq$ under H_0 .
- **Confidence Interval Methods:** Use intervals to test hypotheses.

- **Exponential Family:** Simplifies derivation of tests.
- **Ideal Statistical Method:** Balances simplicity, power, and robustness.

□ 6. Specific Tests

□ Mean Tests

- One-sample, two-sample (paired/unpaired), z or t-tests.

□ Variance Tests

- Chi-square for one sample, F-test for two samples.

□ Multiple Means/Variations

- **ANOVA:** Tests equality of multiple means.
- **Bartlett's Test:** Tests equality of variances.

□ Proportion Tests

- One-sample and two-sample z-tests.

□ Large Sample Tests

- Use normal approximation via CLT.

□ Goodness-of-Fit

- **Chi-square Test:** Compares observed vs expected frequencies.

□ Multinomial Tests

- **Equality of Two Distributions:** Chi-square test.
- **Generalizations:** Extend to more than two groups.

□ Contingency Tables

- **Independence Test:** Chi-square test for association.

□ 7. Sequential Testing

□ SPRT (Sequential Probability Ratio Test)

- **Definition:** Test continues until decision threshold is met.
- **Approximate SPRT:** Uses approximations for practical use.
- **Expected Sample Size:** Depends on true parameter and thresholds.

